



Learning to Notice Algebraically: The Impact of Designed Instructional Material on Student Thinking

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A study investigating changes in students algebraic thinking between 2021-2022 was conducted with $n=53$ Year 7 and 8 students (aged 11-13) in New Zealand. After the introduction of *designed instructional materials* that focused on number properties and equivalence findings show that:

- Designed instructional materials are a promising tool for supporting students to begin noticing algebraic structure, with 86% more students identifying relationships between equivalent equations.
- The associative property of addition and exponents were more likely to be identified and explained by students.
- Identifying the associative and distributive properties of multiplication; and developing generalisations of these remained challenging.

Figure 1 shows an example of a task from the instructional materials. Accompanying each task was information regarding big mathematical ideas, curriculum links, expected learning outcomes and teacher notes that describe algebraic aspects teachers must be aware of noticing and making explicit.

Task 7	Can you work together in your group to solve these number sentences? Make sure that you develop an explanation and justification. $189 + 25 = _ + 26$ $85 - _ = 75 - 28$ $674 + 56 - _ = 671$ $24 \times 16 = 48 \times _$ $105 \div 15 = (45 \div 15) + (_ \div 15)$	Mathematical language	Equivalent, equal sign.
Big Ideas	Equations show relationships of equality between parts on either side of the equal sign. The properties of equality are: If the same real number is added or subtracted to both sides of an equation, equality is maintained; If both sides of an equation are multiplied or divided by the same real number (not dividing by 0), equality is maintained; Two quantities equal to the same third quantity are equal to each other.	Sharing back/Connect	Select student solution strategies that use relational reasoning. Connect: Ask students to generate conjectures related to the equivalence problems that build on the properties of equality.
Curriculum Links	NA4-1: Use a range of multiplicative strategies when operating on whole numbers. NA4-8: Generalise properties of multiplication and division with whole numbers. NA4-7: Form and solve simple linear equations.	Teacher Notes	<ul style="list-style-type: none"> • Students may initially treat the equals sign as an operator or indication to write the answer next. • Students also may compute each side to work out whether they are equal. • Notice students who use the relationships across the equals sign to see whether there is balance. • Highlight to the students to look across the equals sign and find the relationships between numbers to the left and the numbers on the right. • Notice students who use the relationships across the equals sign to see whether there is balance. • Highlight the students relational responses (e.g., noticing the $+ 2 - 2$ relationships). • Press for use of arrows and notations to highlight the relationships.
Learning Outcomes Students will be able to:	Explain and justify relationships between numbers in an equation. Write statements of equivalence in words and using notation. Solve equivalence problems and explain and justify the solutions.		

Figure 1. Example task from the instructional unit (Number and Algebra)

For more information, please refer to the following paper presented at the 45th Annual Conference of MERGA in July 2023. Wadham, B; Pearce, E. & Hunter, J (2023). Learning to notice algebraically: The impact of designed instructional material on student thinking. In Reid-O'Connor, B, Prieto-Rodriguez, Holmes, K & Hughes, A. (Eds.), *Proceedings of the 45th Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 517-524). Newcastle, Australia: The Mathematics Education Group of Australasia Inc.